Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2008

MARKING SCHEME

APPLIED MATHEMATICS

HIGHER LEVEL

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General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2).

2 The marking scheme shows one or more approaches to solving each question. In many cases there are other equally valid methods of solution.

1. **(a)** A ball is thrown vertically upwards with an initial velocity of 39.2 m/s.

Find **(i)** the time taken to reach the maximum height

(ii) the distance travelled in 5 seconds.

(i) $v = u + ft$

$0 = 39.2 - 9.8(t)$

$t = 4$ s — 5

(ii) $s = ut + \frac{1}{2}ft^2$

$= 39.2(4) - 4.9(16)$

$= 78.4$ m — 5

fifth second :

$s = ut + \frac{1}{2}ft^2$

$= 0 + 4.9(1)$

$= 4.9$ m — 5

total distance $= 78.4 + 4.9$

$= 83.3$ m — 5

20

1. **(b)** Two particles P and Q, each having constant acceleration, are moving in the same direction along parallel lines. When P passes Q the speeds are 23 m/s and 5.5 m/s, respectively. Two minutes later Q passes P, and Q is then moving at 65.5 m/s.

Find **(i)** the acceleration of P and the acceleration of Q

(ii) the speed of P when Q overtakes it

(iii) the distance P is ahead of Q when they are moving with equal speeds.

(*i*) Q

$$v = u + ft$$
$$65.5 = 5.5 + f(120)$$
$$f = \frac{1}{2} \text{ m/s}^2 \qquad 5$$
$$s = ut + \tfrac{1}{2} ft^2$$
$$= 5.5(120) + \frac{1}{2}\left(\frac{1}{2}\right)(120)^2$$
$$= 4260 \text{ m} \qquad 5$$

P

$$s = ut + \tfrac{1}{2} ft^2$$
$$4260 = 23(120) + \frac{1}{2}a(120)^2$$
$$a = \frac{5}{24} \text{ m/s}^2 \qquad 5$$

(*ii*)

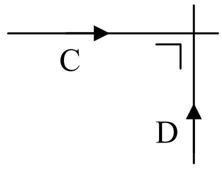
$$v = u + at$$
$$= 23 + \frac{5}{24}(120)$$
$$= 48 \text{ m/s} \qquad 5$$

(*iii*)

$$V_P = V_Q$$
$$23 + \frac{5}{24}t = 5.5 + \frac{1}{2}t \quad \Rightarrow t = 60 \text{ s} \qquad 5$$
$$S_P = 23(60) + \frac{1}{2}\left(\frac{5}{24}\right)(60)^2 = 1755$$
$$S_Q = 5.5(60) + \frac{1}{2}\left(\frac{1}{2}\right)(60)^2 = 1230$$
$$\text{distance} = 1755 - 1230 = 525 \text{ m.} \qquad 5$$

30

2. **(a)** Two straight roads cross at right angles.
A woman C, is walking towards the intersection with a uniform speed of 1.5 m/s.
Another woman D is moving towards the intersection with a uniform speed of 2 m/s.



C is 100 m away from the intersection as D passes the intersection.

Find **(i)** the velocity of C relative D

(ii) the distance of C from the intersection when they are nearest together.

(i) $\vec{V}_C = 1.5\,\vec{i} + 0\,\vec{j}$

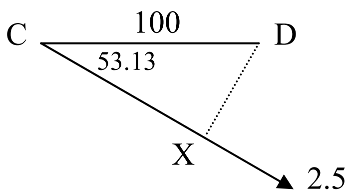
$\vec{V}_D = 0\,\vec{i} + 2\,\vec{j}$

$\vec{V}_{CD} = \vec{V}_C - \vec{V}_D$

$= 1.5\,\vec{i} - 2\,\vec{j}$ 5

magnitude : 2.5 m/s 5

direction : East $53.13°$ South 5



(ii) $$\text{time} = \frac{|CX|}{|\vec{V}_{CD}|}$$

$$= \frac{100\cos 53.13°}{2.5}$$

$= 24$ s 5

In this time C travels $1.5 \times 24 = 36$ m

distance of C from the intersection $= 100 - 36$

$= 64$ m 5

25

2 **(b)** On a particular day the velocity of the wind, in terms of $\vec{i}$ and $\vec{j}$, is $x\,\vec{i}\,-3\,\vec{j}$, where $x \in$ N.
$\vec{i}$ and $\vec{j}$ are unit vectors in the directions East and North respectively.

To a man travelling due East the wind appears to come from a direction North α° West where $\tan\alpha = 2$.

When he travels due North at the same speed as before, the wind appears to come from a direction North β° West where $\tan\beta = \frac{3}{2}$.

Find the actual direction of the wind.

$$\begin{aligned}\vec{V}_M &= a\,\vec{i}\\ \vec{V}_{WM} &= \vec{V}_W - \vec{V}_M\\ &= \left(x\,\vec{i} - 3\,\vec{j}\right) - a\,\vec{i}\\ &= (x-a)\,\vec{i} - 3\,\vec{j}\end{aligned}$$

5

$$\tan\alpha = \frac{x-a}{3}$$

$$2 = \frac{x-a}{3} \quad \Rightarrow a = x-6$$

5

$$\begin{aligned}\vec{V}_M &= a\,\vec{j}\\ \vec{V}_{WM} &= \vec{V}_W - \vec{V}_M\\ &= \left(x\,\vec{i} - 3\,\vec{j}\right) - a\,\vec{j}\\ &= x\,\vec{i} - (3+a)\,\vec{j}\end{aligned}$$

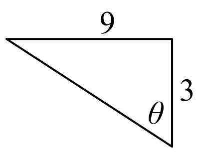
5

$$\tan\beta = \frac{x}{3+a}$$

$$\frac{3}{2} = \frac{x}{3+a} \quad \Rightarrow a = \frac{2x-9}{3}$$

5

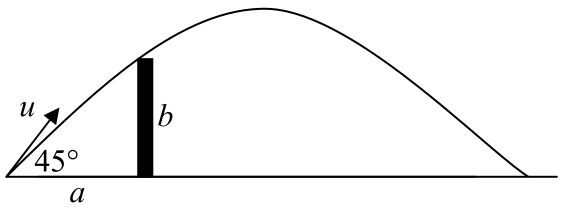
$$\frac{2x-9}{3} = x-6 \quad \Rightarrow x = 9$$



$$\vec{V}_W = 9\,\vec{i} - 3\,\vec{j} \quad \Rightarrow \tan\theta = 3$$

direction of wind: from North 71.6° West 5 | 25

3. **(a)** A ball is projected from a point on the ground at a distance of a from the foot of a vertical wall of height b, the velocity of projection being u at an angle 45° to the horizontal.



If the ball just clears the wall prove that the greatest height reached is

$$\frac{a^2}{4(a-b)}.$$

$$u\cos 45.t = a \qquad 5$$

$$u\sin 45.t - \tfrac{1}{2}gt^2 = b \qquad 5$$

$$u\frac{1}{\sqrt{2}}\left(\frac{a\sqrt{2}}{u}\right) - \tfrac{1}{2}g\left(\frac{a\sqrt{2}}{u}\right)^2 = b$$

$$a - \frac{ga^2}{u^2} = b$$

$$\Rightarrow u^2 = \frac{ga^2}{a-b}$$

At greatest height $v_j = 0$

$$u\frac{1}{\sqrt{2}} - gt = 0$$

$$t = \frac{u}{g\sqrt{2}} \qquad 5$$

$$\text{greatest height} = u\sin 45.t - \tfrac{1}{2}gt^2$$

$$= u\frac{1}{\sqrt{2}}\left(\frac{u}{g\sqrt{2}}\right) - \tfrac{1}{2}g\left(\frac{u}{g\sqrt{2}}\right)^2 \qquad 5$$

$$= \frac{u^2}{4g}$$

$$= \frac{ga^2}{(a-b)4g}$$

$$= \frac{a^2}{4(a-b)} \qquad 5 \qquad 25$$

3 **(b)** A particle is projected down an inclined plane with initial velocity u m/s. The line of projection makes an angle of $2\theta°$ with the inclined plane and the plane is inclined at $\theta°$ to the horizontal.
The plane of projection is vertical and contains the line of greatest slope.

The range of the particle on the inclined plane is $\dfrac{ku^2}{g}\sin\theta$.

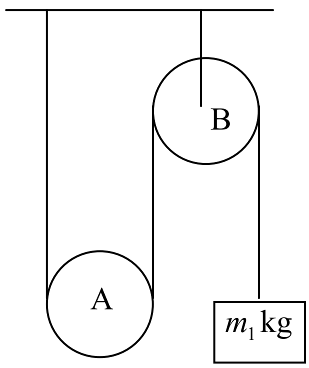
Find the value of k.

$r_j = 0$	5	
$0 = u\sin 2\theta . t - \frac{1}{2} g\cos\theta . t^2$	5	
$\Rightarrow \quad t = \dfrac{2u\sin 2\theta}{g\cos\theta}$ or $\dfrac{4u\sin\theta}{g}$	5	
$\text{Range} = u\cos 2\theta\left\{\dfrac{4u\sin\theta}{g}\right\} + \dfrac{1}{2} g\sin\theta . \left\{\dfrac{4u\sin\theta}{g}\right\}^2$	5	
$= \dfrac{4u^2}{g}\{\cos 2\theta\sin\theta + 2\sin\theta\sin^2\theta\}$		
$= \dfrac{4u^2}{g}\{(\cos^2\theta - \sin^2\theta)\sin\theta + 2\sin^3\theta\}$		
$= \dfrac{4u^2}{g}\{\cos^2\theta\sin\theta + \sin^3\theta\}$		
$= \dfrac{4u^2}{g}\{\sin\theta(\cos^2\theta + \sin^2\theta)\}$		
$= \dfrac{4u^2\sin\theta}{g}$		
$\Rightarrow \quad k = 4$	5	25

4. **(a)** The diagram shows a light inextensible string having one end fixed, passing under a smooth movable pulley A of mass m kg and then over a fixed smooth light pulley B.
The other end of the string is attached to a particle of mass m_1 kg.

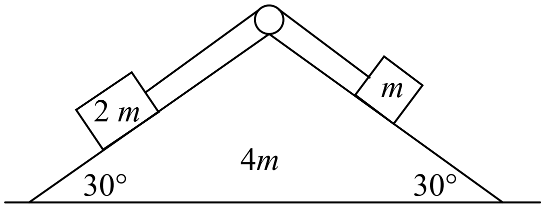
The system is released from rest.

Show that the upward acceleration of A is
$$\frac{(2m_1 - m)g}{4m_1 + m}.$$



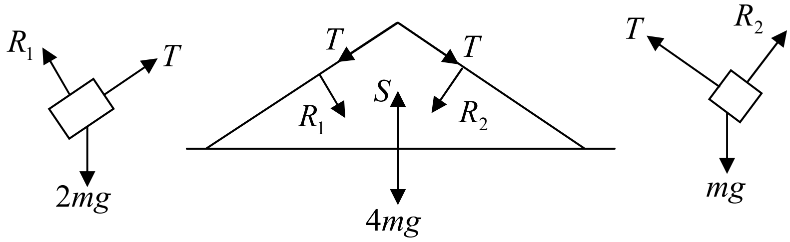
Upward acceleration of A $= f$	5
Downward acceleration of $m_1 = 2f$	5
$m_1 g - T = m_1(2f)$	5
$2T - mg = m(f)$	5
$2m_1 g - 2T = 4m_1 f$ $2T - mg = mf$	
$2m_1 g - mg = (4m_1 + m)f$	5
$f = \dfrac{(2m_1 - m)g}{4m_1 + m}$	5
	30

4 **(b)** Particles of mass $2m$ and m are connected by a light inextensible string which passes over a smooth pulley at the vertex of a wedge-shaped block, one particle resting on each of the smooth faces.
The mass of the wedge is $4m$
and the inclination of each face to the horizontal is 30°.
The wedge rests on a smooth horizontal surface and the system is released from rest.



(i) Show, on separate diagrams, the forces acting on the wedge and on the particles.
(ii) Find the acceleration of the wedge.

(*i*)



5,5

(*ii*) $2m$ $\qquad 2mg\cos 30 - R_1 = 2m(q\sin 30)$

m $\qquad R_2 - mg\cos 30 = m(q\sin 30)$ 5

$$T\cos 30 - T\cos 30 + R_1\sin 30 - R_2\sin 30 = 4mq$$

$$R_1 - R_2 = 8mq$$

$$\{2mg\cos 30 - 2mq\sin 30\} - \{mg\cos 30 + mq\sin 30\} = 8mq$$

$$mg\cos 30 = 8mq + 3mq\sin 30$$

$$g\frac{\sqrt{3}}{2} = 8q + \frac{3}{2}q$$

$$\Rightarrow q = \frac{g\sqrt{3}}{19}$$

5 20

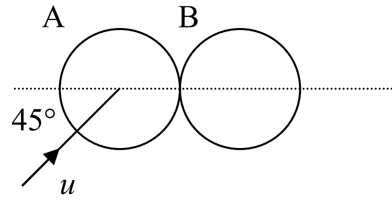
5. **(a)** Three identical smooth spheres lie at rest on a smooth horizontal table with their centres in a straight line. The first sphere is given a speed 2 m/s and it collides directly with the second sphere. The second sphere then collides directly with the third sphere.
The coefficient of restitution for each collision is e, where $e < 1$.

(i) Find, in terms of e, the speed of each sphere after two collisions have taken place.
(ii) Show that there will be at least one more collision.

(*i*) PCM $m(2) + m(0) = mv_1 + mv_2$	5
NEL $v_1 - v_2 = -e(2-0)$	
$v_1 = 1-e$ and $v_2 = 1+e$	5
PCM $m(1+e) + m(0) = mv_3 + mv_4$	5
NEL $v_3 - v_4 = -e(1+e)$	
$v_3 = \frac{1}{2}(1-e^2)$ and $v_4 = \frac{1}{2}(1+e)^2$	5
speeds after 2nd impact : $1-e, \frac{1}{2}(1-e^2), \frac{1}{2}(1+e)^2$	
(*ii*) First sphere will collide : again with second sphere if	
$1-e > \frac{1}{2}(1-e^2)$	
$1-e > \frac{1}{2} - \frac{1}{2}e^2$	
$e^2 - 2e + 1 > 0$	
$(e-1)^2 > 0$	
This is true for $e < 1$	5

25

5 ***(b)*** A smooth sphere A moving with speed u, collides with an identical smooth sphere B which is at rest.



The direction of motion of A, before impact, makes an angle of $45°$ with the line of centres at the instant of impact.

The coefficient of restitution between the spheres is e.

Show that the direction of motion of A is deflected through an angle α where

$$\tan\alpha = \frac{1+e}{3-e}.$$

PCM $\quad m(u\cos 45) + m(0) = mv_1 + mv_2$	5	
NEL $\quad v_1 - v_2 = -e\,(u\cos 45 - 0)$	5	
$\Rightarrow \quad v_1 = \dfrac{u}{2\sqrt{2}}(1-e)$	5	
$\tan(\alpha+45) = \dfrac{u\sin 45}{v_1}$		
$= \dfrac{\frac{u}{\sqrt{2}}}{\frac{u}{2\sqrt{2}}(1-e)}$		
$\dfrac{\tan\alpha+1}{1-\tan\alpha} = \dfrac{2}{1-e}$	5	
$\tan\alpha + 1 - e\tan\alpha - e = 2 - 2\tan\alpha$		
$(3-e)\tan\alpha = 1+e$		
$\tan\alpha = \dfrac{1+e}{3-e}$	5	25

6. **(a)** A particle of mass 5 kg is suspended from a fixed point by a light elastic string which hangs vertically. The elastic constant of the string is 500 N/m.
The mass is pulled down a vertical distance of 20 cm from the equilibrium position and is then released from rest.

(i) Show that the particle moves with simple harmonic motion.
(ii) Find the speed and acceleration of the mass 0.1 seconds after it is released from rest.

(i) Equilibrium position :

$$T_0 = kd \quad \text{and} \quad T_0 = 5g$$

$$\Rightarrow d = \frac{5g}{k} = \frac{5g}{500} \text{ or } \frac{g}{100} \qquad 5$$

Displaced position :

$$\text{Force in dirn. of } x \text{ inc.} = 5g - 500(d+x)$$
$$= 5g - 5g - 500x$$
$$= -500x$$

$$\text{Acceleration} = -\frac{500x}{5} = -100x$$

$$\Rightarrow \text{S.H.M. about } x = 0 \text{ with } \omega = 10 \qquad 5$$

(ii)

$$\text{amplitude} = 0.2$$
$$x = a\cos\omega t$$
$$= 0.2\cos 1 = 0.10806 \qquad 5$$

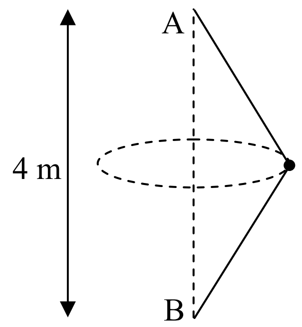
$$v = \omega\sqrt{a^2 - x^2}$$
$$= 10\sqrt{0.2^2 - 0.10806^2}$$
$$= 1.68 \text{ m/s} \qquad 5$$

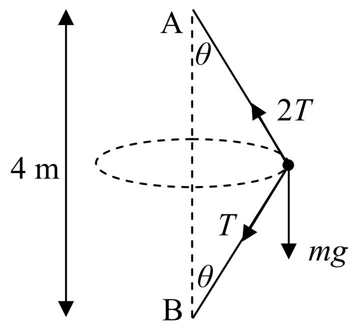
$$\text{acceleration} = \omega^2 x$$
$$= 100(0.10806)$$
$$= 10.8 \text{ m/s}^2 \qquad 5$$

25

6 **(b)** A and B are two fixed pegs, A is 4 m vertically above B.
A mass m kg, connected to A and B by two light inextensible strings of equal length, is describing a horizontal circle with uniform angular velocity ω.

For what value of ω will the tension in the upper string be double the tension in the lower string?





5

$$2T\cos\theta - T\cos\theta = mg$$

5

$$T\cos\theta = mg$$

$$T\left(\frac{2}{\ell}\right) = mg$$

$$T = \frac{mg\ell}{2}$$

5

$$2T\sin\theta + T\sin\theta = mr\omega^2$$

5

$$3T\sin\theta = m\ell\sin\theta\omega^2$$

$$3T = m\ell\omega^2$$

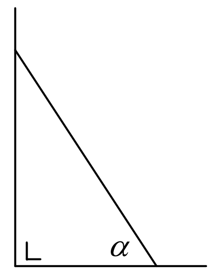
$$3\left(\frac{mg\ell}{2}\right) = m\ell\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{3g}{2}}$$

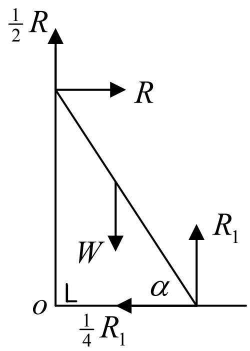
5

25

7. **(a)** One end of a uniform ladder, of weight W, rests against a rough vertical wall, and the other end rests on rough horizontal ground. The coefficient of friction at the ground is $\frac{1}{4}$ and at the wall is $\frac{1}{2}$.
The ladder makes an angle α with the horizontal and is in a vertical plane which is perpendicular to the wall.



The ladder is on the point of slipping.
Find $\tan\alpha$.



$$R = \tfrac{1}{4}R_1$$ 5

$$\tfrac{1}{2}R + R_1 = W$$ 5

Take moments about o for system :

$$R(\ell\sin\alpha) + W\left(\tfrac{1}{2}\ell\cos\alpha\right) =$$
$$= R_1(\ell\cos\alpha)$$ 5

$$R\tan\alpha + \tfrac{1}{2}W = R_1$$

$$\tfrac{1}{4}R_1\tan\alpha + \tfrac{1}{2}\left(\tfrac{1}{8}R_1 + R_1\right) = R_1$$ 5

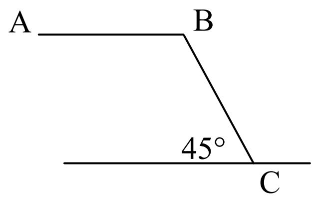
$$\tfrac{1}{4}\tan\alpha + \tfrac{9}{16} = 1$$

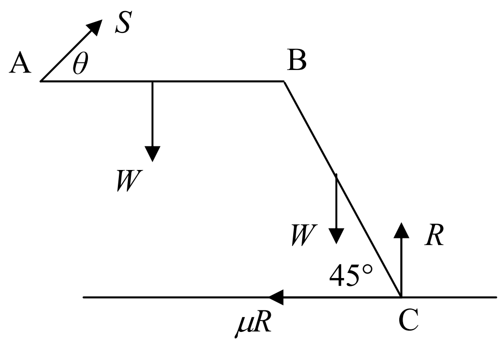
$$\Rightarrow \tan\alpha = \frac{7}{4}$$ 5 25

7 **(b)** Two equal uniform rods AB and BC smoothly jointed at B are in equilibrium with the end C resting on a rough horizontal surface. The end A is held above the surface.

The rod AB is horizontal and the rod BC is inclined at an angle of 45° to the horizontal.

If C is on the point of slipping find the coefficient of friction.





Moments about A for system :

$$W\ell + W\left(2\ell + \frac{\ell}{\sqrt{2}}\right) + \mu R\left(\frac{2\ell}{\sqrt{2}}\right) = \qquad 5$$

$$= R\left(2\ell + \frac{2\ell}{\sqrt{2}}\right) \qquad 5$$

$$3W\sqrt{2} + W + 2\mu R = 2R\sqrt{2} + 2R$$

Moments about B for BC :

$$W\left(\frac{\ell}{\sqrt{2}}\right) + \mu R\left(\frac{2\ell}{\sqrt{2}}\right) = R\left(\frac{2\ell}{\sqrt{2}}\right) \qquad 5$$

$$W + 2\mu R = 2R$$

$$\Rightarrow R = \frac{3W}{2} \qquad 5$$

$$W + 2\mu\left(\frac{3W}{2}\right) = 2\left(\frac{3W}{2}\right)$$

$$\mu = \frac{2}{3} \qquad 5 \qquad 25$$

8. **(a)** Prove that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.

Let $\text{M} = \text{mass per unit area}$

$$\text{mass of element} = \text{M}\{2\pi\,\text{x}\,\text{dx}\}$$

$$\text{moment of inertia of the element} = \text{M}\{2\pi\,\text{x}\,\text{dx}\}\text{x}^2 \qquad 5$$

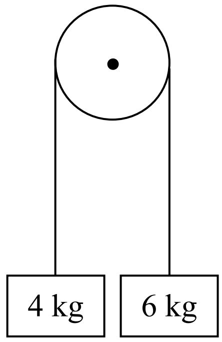
$$\text{moment of inertia of the disc} = \text{M}\,2\pi \int_0^r \text{x}^3\,\text{dx} \qquad 5$$

$$= \text{M}\,2\pi \left[\frac{\text{x}^4}{4}\right]_0^r \qquad 5$$

$$= \tfrac{1}{2}\text{M}\,\pi\,r^4$$

$$= \tfrac{1}{2}\text{m}\,r^2 \qquad 5 \qquad 20$$

8 **(b)** Masses of 4 kg and 6 kg are suspended from the ends of a light inextensible string which passes over a pulley.
The axis of rotation of the pulley is horizontal, perpendicular to the pulley, and passes through the centre of the pulley.
The moment of inertia of the pulley is 0.08 kg m^2 and its radius is 20 cm.
The particles are released from rest and move vertically.
When each mass has acquired a speed of 1 m/s, find



(i) the common acceleration of the masses

(ii) the tensions in the vertical portions of the string.

(i)

$$\text{Gain in KE} = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2$$
$$= \tfrac{1}{2}(0.08)(5^2) + \tfrac{1}{2}(6)(1^2) + \tfrac{1}{2}(4)(1^2)$$
$$= 6$$

5

$$\text{Loss in PE} = 6gh - 4gh$$
$$= 2gh$$

5

$$\text{Gain in KE} = \text{Loss in PE}$$
$$6 = 2gh$$
$$h = \frac{3}{g}$$

5

$$v^2 = u^2 + 2as$$
$$1 = 0 + \frac{6a}{g} \Rightarrow a = \frac{g}{6}$$

5

(ii)

$$6g - T_2 = 6\left(\frac{g}{6}\right) \Rightarrow T_2 = 5g \text{ or } 49 \text{ N}$$

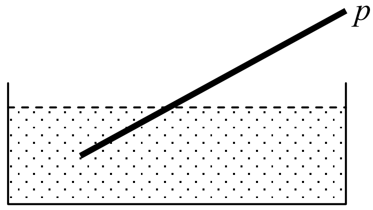
5

$$T_1 - 4g = 4\left(\frac{g}{6}\right) \Rightarrow T_1 = \frac{14g}{3} \text{ or } 45.7 \text{ N}$$

5

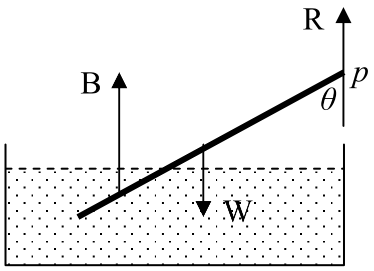
30

9. **(a)** A uniform rod, of length 2 m and relative density $\frac{7}{9}$, is pivoted at one end p and is free to move about a horizontal axis through p. The other end of the rod is immersed in water.



The rod is in equilibrium and is inclined to the vertical as shown in the diagram.

Find the length of the immersed part of the rod.



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Let the length of immersed part $= \ell$

$$B = \frac{\frac{\ell}{2}W}{\frac{7}{9}} \text{ or } \frac{9\ell W}{14}$$ 5

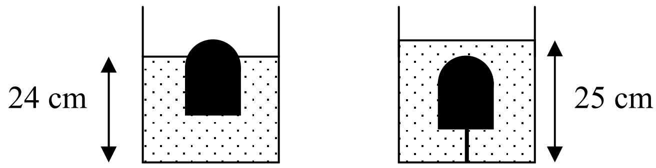
$$B\left(2-\frac{\ell}{2}\right)\sin\theta = W(1)\sin\theta$$ 5

$$\frac{9\ell W}{14}\left(2-\frac{\ell}{2}\right) = W$$

$$9\ell^2 - 36\ell + 28 = 0$$

$$\Rightarrow \ell = 1.06\text{ m.}$$ 5 20

9 **(b)** A cylinder contains water to a height of 20 cm. A solid body of mass 0.06 kg is placed in the cylinder. It floats and the water level rises to 24 cm.



The body is then completely submerged and tied by a string to the bottom of the cylinder. The water level rises to 25 cm.

Find **(i)** the relative density of the body

(ii) the tension in the string

(iii) the radius of the cylinder.

(i)

Let cross - sectional : area of cylinder = A

$$B = W$$
$$(0.04)A(1000)g = (0.05)A(1000s)g$$
$$s = 0.8 \qquad 5$$

(ii)

$$B = T + W \qquad 5$$
$$\frac{0.06g}{0.8} = T + 0.06g \qquad 5$$
$$T = 0.015g \quad \text{or} \quad 0.147\ \text{N} \qquad 5$$

(iii)

$$W = 0.06g$$
$$\rho V g = 0.06g$$
$$800(0.05)Ag = 0.06g$$
$$A = 0.0015 \qquad 5$$

$$\pi r^2 = 0.0015$$
$$r = 0.0218\ \text{m} \quad \text{or} \quad 2.18\ \text{cm}. \qquad 5$$

30

10. **(a)** If

$$x^2 y\frac{dy}{dx} + y\frac{dy}{dx} = 1$$

and $y = 0$ when $x = 0$, find the value of x when $y = \sqrt{\frac{\pi}{2}}$.

$$x^2 y\frac{dy}{dx} + y\frac{dy}{dx} = 1$$

$$y\frac{dy}{dx}\left(1 + x^2\right) = 1$$

$$\int y\,dy = \int \frac{1}{1+x^2}\,dx \qquad 5$$

$$\frac{1}{2}y^2 = \tan^{-1} x + C \qquad 5$$

$$y = 0,\ x = 0 \ \Rightarrow\ C = 0 \qquad 5$$

$$\frac{1}{2}y^2 = \tan^{-1} x$$

$$y = \sqrt{\frac{\pi}{2}}$$

$$\Rightarrow\ \frac{\pi}{4} = \tan^{-1} x$$

$$\Rightarrow\ x = 1 \qquad 5$$

20

10 **(b)** A train of mass 200 tonnes moves along a straight level track against a resistance of $400v^2$, where v m/s is the speed of the train.
The engine exerts a constant power of P kW.

The acceleration of the train is $\dfrac{8000 - v^3}{500v}$.

(i) Find the value of P.

(ii) The train travels a distance 69.07 m while its speed increases from 10 m/s to v_1 m/s. Find the value of v_1.

(*i*)

$$T = \frac{1000P}{v}$$

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$\frac{1000P}{v} - 400v^2 = 200\,000\left(\frac{8000 - v^3}{500v}\right)$$

$$1000P - 400v^3 = 3200\,000 - 400v^3$$

$$P = 3200 \qquad 5$$

(*ii*)

$$v\frac{dv}{ds} = \frac{8000 - v^3}{500v} \qquad 5$$

$$\int_{10}^{v_1} \frac{500v^2}{8000 - v^3}\, dv = \int_0^{69.07} dx \qquad 5$$

$$\left[-\frac{500}{3}\ln\left(8000 - v^3\right)\right]_{10}^{v_1} = [x]_0^{69.07} \qquad 5$$

$$-\frac{500}{3}\ln\left(8000 - {v_1}^3\right) + \frac{500}{3}\ln(7000) = 69.07 \qquad 5$$

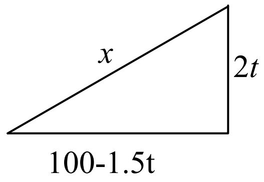
$$\frac{500}{3}\ln\left(\frac{7000}{8000 - {v_1}^3}\right) = 69.07$$

$${v_1}^3 = 3374.936276$$

$$v_1 = 15.0 \text{ m/s} \qquad 5 \qquad 30$$

Some Alternative Solutions

2 (a)



(ii)

$$x^2 = (100-1.5t)^2 + 4t^2$$

$$2x\frac{dx}{dt} = 2(100-1.5t)(-1.5) + 8t$$

$$= 0$$

if $t = 24$ 5

In this time C travels $1.5 \times 24 = 36$ m

distance of C from the intersection $= 100 - 36$

$= 64$ m 5

2 (b)

$$\vec{V}_M = a\,\vec{i}$$

$$\vec{V}_{WM} = \frac{2v}{\sqrt{5}}\,\vec{i} - \frac{v}{\sqrt{5}}\,\vec{j}$$

$$\vec{V}_W = \vec{V}_{WM} + \vec{V}_M$$

$$= \left(\frac{2v}{\sqrt{5}} + a\right)\vec{i} - \frac{v}{\sqrt{5}}\,\vec{j}$$ 5

$$= x\,\vec{i} - 3\,\vec{j}$$

$$\Rightarrow v = 3\sqrt{5} \quad \text{and} \quad a = x - 6$$ 5

$$\vec{V}_M = a\,\vec{j}$$

$$\vec{V}_{WM} = \frac{3w}{\sqrt{13}}\,\vec{i} - \frac{2w}{\sqrt{13}}\,\vec{j}$$

$$\vec{V}_W = \vec{V}_{WM} + \vec{V}_M$$

$$= \left(\frac{3w}{\sqrt{13}}\right)\vec{i} + \left(a - \frac{2w}{\sqrt{13}}\right)\vec{j}$$ 5

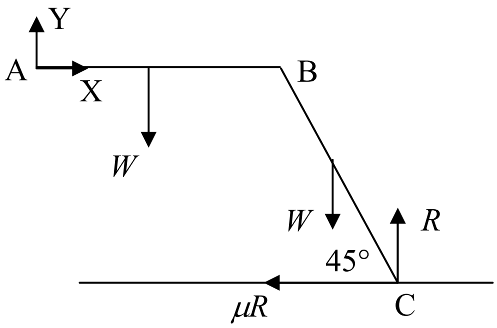
$$= x\,\vec{i} - 3\,\vec{j}$$

$$\Rightarrow w = \frac{(a+3)\sqrt{13}}{2} \quad \text{and} \quad a = \frac{2x-9}{3}$$ 5

5 (b)

PCM $\quad m(u\cos 45) + m(0) = mv_1 + mv_2$	5	
NEL $\quad v_1 - v_2 = -e\,(u\cos 45 - 0)$	5	
$\Rightarrow \quad v_1 = \dfrac{u}{2\sqrt{2}}(1-e)$	5	
$\tan\theta = \dfrac{\frac{u}{\sqrt{2}}}{\frac{u}{2\sqrt{2}}(1-e)} = \dfrac{2}{1-e}$		
$\tan\alpha = \tan(\theta - 45) = \dfrac{\tan\theta - 1}{1+\tan\theta}$		
$\tan\alpha = \dfrac{\frac{2}{1-e} - 1}{1 + \frac{2}{1-e}}$	5	
$\tan\alpha = \dfrac{1+e}{3-e}$	5	25

7 (b)



Moments about C for system :

$$X\left(\frac{2\ell}{\sqrt{2}}\right)+Y\left(2\ell+\frac{2\ell}{\sqrt{2}}\right)=$$

5

$$=W\left(\ell+\frac{2\ell}{\sqrt{2}}\right)+W\left(\frac{\ell}{\sqrt{2}}\right)$$

5

$$2X+2\sqrt{2}Y+2Y=W\sqrt{2}+3W$$

Moments about B for AB :

$$Y(2\ell)=W(\ell)$$

5

$$Y=\frac{W}{2}$$

$$\Rightarrow X=W$$

$$Y+R=2W\Rightarrow R=\frac{3W}{2}$$

5

$$X=\mu R$$

$$X=\mu\left(\frac{3W}{2}\right)$$

$$\Rightarrow \mu=\frac{2}{3}$$

5

25

8 (b)

(i)

$$v = 0.2\omega$$

$$\frac{dv}{dt} = 0.2\frac{d\omega}{dt}$$

$$a = 0.2\ddot{\theta}$$ 5

$$I\ddot{\theta} = T\text{orque}$$

$$0.08\left(\frac{a}{0.2}\right) = (T_2 - T_1)\ 0.2$$ 5

$$2a = T_2 - T_1$$ 5

$$6g - T_2 = 6a$$

$$T_1 - 4g = 4a$$

$$2g - (T_2 - T_1) = 10a$$

$$2g - (2a) = 10a$$

$$a = \frac{g}{6}$$ 5

(ii)

$$6g - T_2 = 6\left(\frac{g}{6}\right) \Rightarrow T_2 = 5g \text{ or } 49\text{ N}$$ 5

$$T_1 - 4g = 4\left(\frac{g}{6}\right) \Rightarrow T_1 = \frac{14g}{3} \text{ or } 45.7\text{ N}$$ 5

30